

A New Model of Agegraphic Dark Energy

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ABSTRACT

In this note, we propose a new model of agegraphic dark energy based on the Károlyházy relation, where the time scale is chosen to be the conformal time η of the Friedmann-Robertson-Walker (FRW) universe. We find that in the radiation-dominated epoch, the equation-of-state parameter of the new agegraphic dark energy $w_q = -1/3$ whereas $\Omega_q = n^2 a^2$; in the matter-dominated epoch, $w_q = -2/3$ whereas $\Omega_q = n^2 a^2/4$; eventually, the new agegraphic dark energy dominates; in the late time $w_q \rightarrow -1$ when $a \rightarrow \infty$, and the new agegraphic dark energy mimics a cosmological constant. In every stage, all things are consistent. The confusion in the original agegraphic dark energy model proposed in arXiv:0707.4049 disappears in this new model. Furthermore, $\Omega_q \ll 1$ is naturally satisfied in both radiation-dominated and matter-dominated epochs where $a \ll 1$. In addition, we further extend the new agegraphic dark energy model by including the interaction between the new agegraphic dark energy and background matter. In this case, we find that w_q can cross the phantom divide.

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I. INTRODUCTION

The cosmological constant problem is essentially a problem in quantum gravity, since the cosmological constant is commonly considered as the vacuum expectation value of some quantum fields. Before a completely successful quantum theory of gravity is available, it is more realistic to consider some consequences of combining quantum mechanics with general relativity directly.

In general relativity, one can measure the spacetime without any limit of accuracy. In quantum mechanics, however, the well-known Heisenberg uncertainty relation puts a limit of accuracy in these measurements. Following the line of quantum fluctuations of spacetime, Károlyházy and his collaborators [1] (see also [2]) made an interesting observation concerning the distance measurement for Minkowski spacetime through a light-clock *Gedanken experiment*, namely, the distance t in Minkowski spacetime cannot be known to a better accuracy than

$$\delta t = \lambda t_p^{2/3} t^{1/3}, \quad (1)$$

where λ is a dimensionless constant of order unity. We use the units $\hbar = c = k_B = 1$ throughout this work. Thus, one can use the terms like length and time interchangeably, whereas $l_p = t_p = 1/m_p$ with l_p , t_p and m_p being the reduced Planck length, time and mass respectively.

The Károlyházy relation (1) together with the time-energy uncertainty relation enables one to estimate a quantum energy density of the metric fluctuations of Minkowski spacetime [2, 3]. Following [2, 3], with respect to Eq. (1) a length scale t can be known with a maximum precision δt determining thereby a minimal detectable cell $\delta t^3 \sim t_p^2 t$ over a spatial region t^3 . Such a cell represents a minimal detectable unit of spacetime over a given length scale t . If the age of the Minkowski spacetime is t , then over a spatial region with linear size t (determining the maximal observable patch) there exists a minimal cell δt^3 the energy of which due to time-energy uncertainty relation cannot be smaller than [2, 3]

$$E_{\delta t^3} \sim t^{-1}. \quad (2)$$

Therefore, the energy density of metric fluctuations of Minkowski spacetime is given by [2, 3]

$$\rho_q \sim \frac{E_{\delta t^3}}{\delta t^3} \sim \frac{1}{t_p^2 t^2} \sim \frac{m_p^2}{t^2}. \quad (3)$$

We refer to the original papers [2, 3] for more details. It is worth noting that in fact, the Károlyházy relation (1) and the corresponding energy density (3) have been independently rediscovered later for many times in the literature (see e.g. [4, 5, 6]).

In [3] (see also [7]), it is noticed that the Károlyházy relation (1) naturally obeys the holographic black hole entropy bound. In fact, the holographic dark energy [8] also stems from the idea of holographic black hole entropy bound [9]. For a complete list of references concerning the holographic dark energy, one can see e.g. [10] and references therein. It is worth noting that the form of energy density Eq. (3) is similar to the one of holographic dark energy [8, 9, 10, 11, 12], i.e., $\rho_\Lambda \sim l_p^{-2} t^{-2}$. The similarity between ρ_q and ρ_Λ might reveal some universal features of quantum gravity, although they arise from different ways. See [7] for a detailed discussion on this point.

In the next section, we briefly review the original agegraphic dark energy model proposed in [7]. The difficulties of this original agegraphic dark energy model are also discussed. In Sec. III, we propose a new model of agegraphic dark energy and find that the confusion in the original agegraphic dark energy model does not exist in this new model. In Sec. IV, we further extend the new agegraphic dark energy model by including the interaction between the new agegraphic dark energy and the background matter. Some concluding remarks are given in Sec. V.

II. THE ORIGINAL AGEGRAPHIC DARK ENERGY MODEL

Based on the energy density (3), a so-called agegraphic dark energy model was proposed in [7]. There, as the most natural choice, the time scale t in Eq. (3) is chosen to be the age of the universe

$$T = \int_0^a \frac{da}{Ha}, \quad (4)$$

where a is the scale factor of our universe; $H \equiv \dot{a}/a$ is the Hubble parameter; a dot denotes the derivative with respect to cosmic time. Thus, the energy density of the agegraphic dark energy is given by [7]

$$\rho_q = \frac{3n^2 m_p^2}{T^2}, \quad (5)$$

where the numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved spacetime (since the energy density is derived for Minkowski spacetime), and so on. Obviously, since the present age of the universe $T_0 \sim H_0^{-1}$ (the subscript “0” indicates the present value of the corresponding quantity; we set $a_0 = 1$), the present energy density of the agegraphic dark energy explicitly meets the observed value naturally, provided that the numerical factor n is of order unity. In addition, by choosing the age of the universe rather than the future event horizon as the length measure, the drawback concerning causality in the holographic dark energy model [8] does not exist in the agegraphic dark energy model [7].

If we consider a flat Friedmann-Robertson-Walker (FRW) universe containing the agegraphic dark energy and pressureless matter, the corresponding Friedmann equation reads

$$H^2 = \frac{1}{3m_p^2} (\rho_m + \rho_q). \quad (6)$$

It is convenient to introduce the fractional energy densities $\Omega_i \equiv \rho_i/(3m_p^2 H^2)$ for $i = m$ and q . From Eq. (5), it is easy to find that

$$\Omega_q = \frac{n^2}{H^2 T^2}, \quad (7)$$

whereas $\Omega_m = 1 - \Omega_q$ from Eq. (6). By using Eqs. (5)–(7) and the energy conservation equation $\dot{\rho}_m + 3H\rho_m = 0$, we obtain the equation of motion for Ω_q as [7]

$$\Omega'_q = \Omega_q (1 - \Omega_q) \left(3 - \frac{2}{n} \sqrt{\Omega_q} \right), \quad (8)$$

where a prime denotes the derivative with respect to the e -folding time $N \equiv \ln a$. From the energy conservation equation $\dot{\rho}_q + 3H(\rho_q + p_q) = 0$, as well as Eqs. (5) and (7), it is easy to find that the equation-of-state parameter (EoS) of the agegraphic dark energy $w_q \equiv p_q/\rho_q$ is given by [7]

$$w_q = -1 + \frac{2}{3n} \sqrt{\Omega_q}. \quad (9)$$

The agegraphic dark energy has some interesting features. From Eq. (9), it is easy to see that $w_q \rightarrow -1$ in the early time where $\Omega_q \rightarrow 0$, whereas $w_q \rightarrow -1 + 2/(3n)$ in the late time where $\Omega_q \rightarrow 1$. On the other hand, if one considers the matter-dominated epoch where $\Omega_q \ll 1$ while $a \ll 1$, Eq. (8) approximately becomes $\Omega'_q \simeq 3\Omega_q$. Therefore, $\Omega_q \propto a^3$ in the matter-dominated epoch. This is consistent with the fact that in the matter-dominated epoch the agegraphic dark energy mimics a cosmological constant, $w_q \simeq -1$, whereas the energy density of pressureless matter scales as $\rho_m \propto a^{-3}$.

However, there is an implicit confusion in this agegraphic dark energy model [13]. Actually, in the matter-dominated epoch with $\Omega_q \ll 1$, one has $a \propto t^{2/3}$, thus $T^2 \propto a^3$. From Eq. (5), in this epoch $\rho_q \propto a^{-3}$. Since $\rho_m \propto a^{-3}$, one has $\Omega_q \simeq \text{const.}$, which is conflict with $\Omega_q \propto a^3$ obtained previously. In addition, from Eq. (5), the agegraphic dark energy tracks the dominated components (either pressureless matter or radiation). Therefore, the agegraphic dark energy never dominates. This is of course unacceptable.

To get around this confusion, there are two different solutions. The first one is to replace T with $T + \delta$, where δ is a constant with dimension of time. Thus, Eq. (5) becomes $\rho_q = 3n^2 m_p^2 (T + \delta)^{-2}$. In the early time, $T \ll \delta$, thus $\rho_q \simeq \text{const.}$, i.e., the agegraphic dark energy mimics a cosmological constant. In the late time, $T \gg \delta$, thus ρ_q can be described approximately by Eq. (5). In the intermediate stage $T \sim \delta$, δ cannot be neglected. So, the tracking behavior no longer exists. However, this solution abandons the motivation of the Károlyházy relation (1), and then becomes only a phenomenological model.

The second solution argues that Eq. (5) was derived for the Minkowski spacetime and is not valid exactly in the early time where the spacetime is highly curved. Thus, $n = n(T)$ may be variable in the early time, say, $n \propto T$. Only after a critical T_c , the factor n can be approximated to a constant in Eq. (5). One can choose T_c in the early or middle stage of the matter-dominated epoch. For $T > T_c$, the energy density of agegraphic dark energy cannot be ignored already (nb. $\Omega_q \sim 0.1 - 0.2$ in the matter-dominated epoch does not violate the constraint from the BBN [14]). Therefore, the tracking behavior no longer exists, and then the confusion disappears. However, this solution is successful at the price of giving up the validity of Eq. (5) in the early time.

III. A NEW MODEL OF AGEGRAPHIC DARK ENERGY

In this note, we seek another more comfortable solution to the confusion mentioned above. The history always repeats itself (“there is nothing new under the sun”, Ecclesiastes 1:9). It is suggestive to review the history of the holographic dark energy model [8, 9, 10, 11, 12]. In [9], Cohen *et al.* argued that $\rho_\Lambda = 3c^2 m_p^2 L^{-2}$ from the holographic principle. The most natural choice of L is the Hubble horizon H^{-1} . However, Hsu [11] (see also [8]) pointed out that in this case the EoS of holographic dark energy is zero and the expansion of the universe cannot be accelerated. The next choice of L is the particle horizon. Unfortunately, in this case, the EoS of holographic dark energy is always larger than $-1/3$ and the expansion of the universe also cannot be accelerated [8]. Finally, Li [8] found out that L might be the future event horizon of the universe. In this case, the EoS of holographic dark energy is given by [8, 12]

$$w_\Lambda = -\frac{1}{3} - \frac{2}{3c} \sqrt{\Omega_\Lambda}. \quad (10)$$

Obviously, $w_\Lambda < -1/3$ and the expansion of the universe can indeed be accelerated. In fact, this is just the holographic dark energy model investigated extensively in the literature, although it has the drawback concerning causality, because the existence of the future event horizon requires an eternal accelerated expansion of the universe.

The development of the holographic dark energy model might shed some light on the agegraphic dark energy model. Can we have another choice of the time scale in Eq. (3) rather than T , the age of the universe? The answer is yes. Here we choose the time scale to be the conformal time η instead, which is defined by $dt = a d\eta$ [where t is the cosmic time, do not confuse it with the t in Eq. (3)]. Notice that the Károlyházy relation (1) is derived for Minkowski spacetime $ds^2 = dt^2 - d\mathbf{x}^2$ [1, 2, 3]. For the FRW universe, $ds^2 = dt^2 - a^2 d\mathbf{x}^2 = a^2(d\eta^2 - d\mathbf{x}^2)$. Thus, it might be more reasonable to choose the time scale in Eq. (3) to be the conformal time η since it is the causal time in the Penrose diagram of the FRW universe [17]. Then, we propose the new agegraphic dark energy as

$$\rho_q = \frac{3n^2 m_p^2}{\eta^2}, \quad (11)$$

where the conformal time

$$\eta \equiv \int \frac{dt}{a} = \int \frac{da}{a^2 H}. \quad (12)$$

If we write η to be a definite integral, there will be an additional integration constant. Thus, $\dot{\eta} = 1/a$. The corresponding fractional energy density is given by

$$\Omega_q = \frac{n^2}{H^2 \eta^2}. \quad (13)$$

At first, we consider a flat FRW universe containing the new agegraphic dark energy and pressureless matter. By using Eqs. (6), (13), (11) and the energy conservation equation $\dot{\rho}_m + 3H\rho_m = 0$, we find that the equation of motion for Ω_q is given by

$$\frac{d\Omega_q}{da} = \frac{\Omega_q}{a} (1 - \Omega_q) \left(3 - \frac{2}{n} \frac{\sqrt{\Omega_q}}{a} \right). \quad (14)$$

From the energy conservation equation $\dot{\rho}_q + 3H(\rho_q + p_q) = 0$, as well as Eqs. (13) and (11), it is easy to find that the EoS of new agegraphic dark energy $w_q \equiv p_q/\rho_q$ is given by

$$w_q = -1 + \frac{2}{3n} \frac{\sqrt{\Omega_q}}{a}. \quad (15)$$

Comparing with Eqs. (8) and (9), the scale factor a enters Eqs. (14) and (15) explicitly. When $a \rightarrow \infty$, $\Omega_q \rightarrow 1$, thus $w_q \rightarrow -1$ in the late time. When $a \rightarrow 0$, $\Omega_q \rightarrow 0$, we cannot obtain w_q from Eq. (15) directly. Let us consider the matter-dominated epoch, $H^2 \propto \rho_m \propto a^{-3}$. Thus, $a^{1/2}da \propto dt = ad\eta$. Therefore, $\eta \propto a^{1/2}$. From Eq. (11), $\rho_q \propto a^{-1}$. From the energy conservation equation $\dot{\rho}_q + 3H\rho_q(1 + w_q) = 0$, we obtain that $w_q = -2/3$. Since $\rho_m \propto a^{-3}$, it is expected that $\Omega_q \propto a^2$. Comparing $w_q = -2/3$ with Eq. (15), we find that $\Omega_q = n^2 a^2/4$ in the matter-dominated epoch as expected. For $a \ll 1$, if n is of order unity, $\Omega_q \ll 1$ naturally. On the other hand, one can check that $\Omega_q = n^2 a^2/4$ satisfies

$$\frac{d\Omega_q}{da} = \frac{\Omega_q}{a} \left(3 - \frac{2}{n} \frac{\sqrt{\Omega_q}}{a} \right),$$

which is the approximation of Eq. (14) for $1 - \Omega_q \simeq 1$. So, all things are consistent. The confusion in the original agegraphic dark energy model [7] does not exist in this new model.

In fact, we can extend our discussion to include the radiation-dominated epoch. To be general, we consider a flat FRW universe containing the new agegraphic dark energy and background matter whose EoS is constant w_m . In particular, $w_m = 0$ for pressureless matter, whereas $w_m = 1/3$ for radiation. By using Eqs. (6), (13), (11) and the energy conservation equation $\dot{\rho}_m + 3H\rho_m(1 + w_m) = 0$, we find that the equation of motion for Ω_q is given by

$$\frac{d\Omega_q}{da} = \frac{\Omega_q}{a} (1 - \Omega_q) \left[3(1 + w_m) - \frac{2}{n} \frac{\sqrt{\Omega_q}}{a} \right]. \quad (16)$$

On the other hand, the EoS of new agegraphic dark energy is the same one given in Eq. (15). In the radiation-dominated epoch, $H^2 \propto \rho_r \propto a^{-4}$. Thus, $ada \propto dt = ad\eta$. Therefore, $\eta \propto a$. From Eq. (11), $\rho_q \propto a^{-2}$. From the energy conservation equation $\dot{\rho}_q + 3H\rho_q(1 + w_q) = 0$, we obtain that $w_q = -1/3$ in the radiation-dominated epoch. Since $\rho_r \propto a^{-4}$, it is expected that $\Omega_q \propto a^2$. Comparing $w_q = -1/3$ with Eq. (15), we find that $\Omega_q = n^2 a^2$ in the radiation-dominated epoch as expected. For $a \ll 1$, if n is of order unity, $\Omega_q \ll 1$ naturally. On the other hand, one can check that $\Omega_q = n^2 a^2$ satisfies

$$\frac{d\Omega_q}{da} = \frac{\Omega_q}{a} \left(4 - \frac{2}{n} \frac{\sqrt{\Omega_q}}{a} \right),$$

which is the approximation of Eq. (16) for $1 - \Omega_q \simeq 1$ and $w_m = 1/3$. Once again, we see that all things are consistent in the radiation-dominated epoch.

In summary, in the radiation-dominated epoch, $w_q = -1/3$ whereas $\Omega_q = n^2 a^2$; in the matter-dominated epoch, $w_q = -2/3$ whereas $\Omega_q = n^2 a^2/4$; eventually, the new agegraphic dark energy dominates; in the late time $w_q \rightarrow -1$ when $a \rightarrow \infty$, the new agegraphic dark energy mimics a cosmological constant. It is worth noting that $\Omega_q \ll 1$ naturally in both radiation-dominated and matter-dominated epochs where $a \ll 1$.

It is easy to see that the behavior of new agegraphic dark energy is very different from the one of original agegraphic dark energy proposed in [7]. In addition, we notice that the evolution behavior of new agegraphic dark energy is similar to that of holographic dark energy [8, 9, 10, 11, 12]. From Eq. (10), the EoS of holographic dark energy $w_\Lambda \simeq -1/3$ in the radiation-dominated and matter-dominated epochs where $\Omega_\Lambda \ll 1$, whereas $w_\Lambda \rightarrow -1/3 - 2/(3c)$ in the late time $a \rightarrow \infty$ where $\Omega_\Lambda \rightarrow 1$. If $c = 1$, the holographic dark energy mimics a cosmological constant in the late time. However, for the new agegraphic dark energy, $w_q = -2/3$ in the matter-dominated epoch; $w_q \rightarrow -1$ in the late time regardless of the value of n . As a result, there also exist some essential differences between the new agegraphic dark energy and the holographic dark energy.

IV. THE NEW AGEGRAPHIC DARK ENERGY MODEL WITH INTERACTION

Following [15, 16], we further extend the new agegraphic dark energy model by including the interaction between the new agegraphic dark energy and background matter whose EoS is constant w_m . We assume that the new agegraphic dark energy and background matter exchange energy through an interaction term Q as

$$\dot{\rho}_q + 3H\rho_q(1 + w_q) = -Q, \quad (17)$$

$$\dot{\rho}_m + 3H\rho_m(1 + w_m) = Q. \quad (18)$$

In this way the total energy conservation equation $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0$ is still kept. In this case, by using Eqs. (6), (13), (11) and (18), we find that the equation of motion for Ω_q is changed to

$$\frac{d\Omega_q}{da} = \frac{\Omega_q}{a} \left\{ (1 - \Omega_q) \left[3(1 + w_m) - \frac{2}{n} \frac{\sqrt{\Omega_q}}{a} \right] - \frac{Q}{3m_p^2 H^3} \right\}. \quad (19)$$

From Eqs. (17), (13) and (11), we obtain the EoS of new agegraphic dark energy

$$w_q = -1 + \frac{2}{3n} \frac{\sqrt{\Omega_q}}{a} - \frac{Q}{3H\rho_q}. \quad (20)$$

It is easy to see that Eqs. (19) and (20) reduce to Eqs. (16) and (15) in the case of $Q = 0$ (i.e. without interaction). It is worth noting that from Eq. (15) w_q is always larger than -1 and cannot cross the phantom divide $w = -1$ in the case of $Q = 0$ (i.e. without interaction). Here, the situation is changed by the interaction $Q \neq 0$. If $Q > 0$, from Eq. (20), one can see that w_q can be smaller than -1 or larger than -1 . Thus, it becomes possible that w_q crosses the phantom divide.

V. CONCLUDING REMARKS

In this note, we propose a new model of agegraphic dark energy based on the Károlyházy relation (1), where the time scale is chosen to be the conformal time η of the FRW universe. We find in this model that in the radiation-dominated epoch, the EoS of new agegraphic dark energy $w_q = -1/3$ whereas $\Omega_q = n^2 a^2$; in the matter-dominated epoch, $w_q = -2/3$ whereas $\Omega_q = n^2 a^2/4$; eventually, the new agegraphic dark energy dominates; in the late time $w_q \rightarrow -1$ when $a \rightarrow \infty$, and the new agegraphic dark energy mimics a cosmological constant. In every stage, all things are consistent. The confusion in the original agegraphic dark energy model proposed in [7] disappears in this model. Furthermore, $\Omega_q \ll 1$ naturally occurs in both radiation-dominated and matter-dominated epochs where $a \ll 1$. In addition, we further study the new agegraphic dark energy model by including the interaction between the new agegraphic dark energy and background matter. In this case, we find that w_q can cross the phantom divide.

The evolution behavior of the new agegraphic dark energy is very different from that of original agegraphic dark energy proposed in [7]. Instead the evolution behavior of the new agegraphic dark energy is similar to that of the holographic dark energy [8, 9, 10, 11, 12]. But, as mentioned above, some essential differences exist between them. In particular, the new agegraphic dark energy model is free of the drawback concerning causality problem which exists in the holographic dark energy model [8]. Another advantage of the new agegraphic dark energy model is that the Károlyházy relation (1) naturally obeys the holographic black hole entropy bound [3, 7].

Furthermore, thanks to its special analytic features in the radiation-dominated and matter-dominated epochs, this new agegraphic dark energy model is actually a *single-parameter* model [18]. To our knowledge, it is the third *single-parameter* cosmological model besides the well-known Λ CDM model and the DGP braneworld model [19]. Also, it is found that the coincidence problem can be solved naturally in this model [18]. In addition, as shown in [18], the new agegraphic dark energy model can fit the cosmological observations of type Ia supernovae, cosmic microwave background, and large scale structure very well. In this sense, it is expected that the new agegraphic dark energy model is a cosmologically viable model for dark energy.

Finally, it is worth noting that there are still some problems in the (new) agegraphic dark energy model. After the appearance of our relevant works on the (new) agegraphic dark energy, it is found that the original agegraphic dark energy model proposed in [7, 15, 16] is difficult to reconcile with the big bang nucleosynthesis (BBN) constraint [20]. On the other hand, as shown in [21], the situation is better in the new agegraphic dark energy model. To alleviate the difficulty from BBN constraint, one can introduce the coupling between the dark matter and dark energy (for references on this topic, see e.g. [15, 16, 22] and references therein). In addition, the (new) agegraphic dark energy model faces the problem of instabilities [23], while the holographic dark energy model also faces the same problem [24]. Nevertheless, we consider that the new agegraphic dark energy model deserves further investigations, since it has some valuable advantages mentioned above. We hope that it can shed new light on the understanding of the mysterious dark energy.

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